REVIEWS

Nonlinear Waves, Solitons and Chaos. By E. INFELD and G. ROWLANDS. Cambridge University Press, 1990. 423 pp. £45 or $85 (hardback), £17.50 or $29.95 (paperback).

The title of this book covers areas which have developed rapidly in recent years and which still excite many mathematicians, engineers and physicists. The authors are both physicists and aim to allow 'the more ambitious reader to get into the field', and to permit a science graduate or senior undergraduate to learn about these new developments. The areas of water waves and waves in plasmas dominate the book, but references to other applications of the mathematical content are frequent.

After a wide-ranging introductory chapter there are three chapters on linear waves and instabilities. Chapter 2 introduces the dispersion equation for some of the simplest waves in plasmas and discusses solution of the Vlasov equation for the distribution function of a plasma and Landau damping. The usual linear wave concepts are briefly introduced. Chapter 3 extends the discussion of group velocity to unstable media and describes both convective and non-convective instabilities. Chapter 4, on surface waves, includes plasma and solid–liquid surfaces as well as fluid interfaces. Much emphasis is given to instabilities especially in a first look at wave–wave interactions which uses the Doppler relation for short waves on long periodic waves.

Weakly nonlinear approximations are introduced in chapter 5 with particular emphasis on modulated waves and the way in which the group velocity of linear waves splits into two characteristic velocities for nonlinear waves. The study of steadily propagating solutions, such as solitary waves, is delayed to chapter 6 which also includes examples where a Lagrangian coordinate approach is used to give some explicit solutions.

A variety of soliton and multi-soliton solutions are introduced in chapter 7. Methods of derivation which are outlined include Hirota's method, inverse scattering, Bäcklund transformations and the trace method. Attention is not restricted to simple examples, e.g. the two-soliton solutions, in (x, y, t), of the Kadomtsev–Petviashvili equations, and the way in which Lax pairs generate equations and solutions are discussed.

Chapter 8 is the climax of the book where much of the preceding matter is put in perspective with consideration of the evolution and instability of initially one-dimensional waves. Most of this is based on Whitham's method of averaging and related methods including the authors' small-wavenumber modulation expansion. There is further discussion of the concept of group velocity which is not simply and uniquely defined for nonlinear waves. Due note is taken of the fact that one-dimensional solutions are descriptions of waves in a physical space of three dimensions. There are general discussions and indications of possible future research.

Cylindrical and spherical solitons are the topic of chapter 9, where, naturally, a greater variety of behaviour can be found in plasmas than on a water surface.

Chaos appears only in the final chapter, which has a relatively detailed description of the behaviour of the quadratic, logistic iteration. Strange attractors, Poincaré maps for ordinary differential equations, Lyapunov number and dimensions are other topics in this rather isolated chapter.

The style of presentation is to treat many topics briefly, with a few at greater length, and to lead the reader to relevant references. There are 21 pages of them, totalling around 500 references of which a high proportion are later than 1970. Even so, such is the level of activity in this area that numerous useful references are missing, for example, Craik's (1985) book, and many papers in this journal which is only referred to 16 times.

In such a rapidly growing field the choice of possible topics is wide and many must be omitted. The most unfortunate omission is the concept of wave-action, which since its introduction by Bretherton & Garrett (1968) has proved invaluable. Another omission is any discussion of hyperbolic nonlinear waves with their steepening solutions and shocks or jumps. It seems that the authors implicitly expect the reader to be familiar with Lighthill's (1978) *Waves in Fluids*. This is a pity since they are clearly trying to refine physical concepts from the avalanche of mathematical results, and the simplest examples may be the most valuable. On the other hand this could be a blind-spot in the authors' appreciation of nonlinear waves. They discuss characteristics of modulations on finite-amplitude waves without mention of the wave jumps that such characteristics can lead to (e.g. Peregrine 1983, 1985).

Despite these omissions I think the authors have succeeded in their aim to lead a graduate or ambitious reader into the field. Even someone familiar with nonlinear waves is likely to find that they are led to unfamiliar yet useful and interesting works. On the other hand I do not recommend a senior undergraduate to be given this book. There are so many typographical mistakes in the mathematical formulae that a newcomer should be prepared to check all details. Unfortunately even the first statement of the Korteweg–de Vries equation contains a misprint. Further, many readers may take exception to the authors' use of 'soliton' in a wide sense to describe any isolated wave, rather than in its more usual sense to denote waves that interact with each other and yet emerge unchanged.

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D. H. Peregrine

Handbook of Hydraulic Engineering. By A. Lencastre. Ellis Horwood, 1987. 540 pp. £69.50.

This book is a translation into English (by P. Holmes) of the author's popular *Manual of Hydraulics*, the first edition of which was published in Portugal in 1957. Since that time the work has undergone many revisions and improvements so that it now contains an abundance of practical information on hydraulic design. Also, in preparing the English edition, it was decided to omit some of the theoretical sections since these were well covered by the many fluid mechanics textbooks available in English.

The principal attraction therefore of this work is the abundant information it contains on all the practical aspects of hydraulic design. After dealing with the physical properties of fluids and the theoretical bases of the science of hydraulics, it goes on to cover flow in pipes, open channel flow, flow in porous media, flow measurement, pumps and transient flow in conduits.

Each of the ten chapters is set out in a logical way, with fundamental matters treated first and special cases dealt with at the end. A vital part of the work is the very detailed collection of tables and graphs which accompany the text. In fact this data abstraction section amounts to about 40% of the work, extending to nearly 200 pages. Examples of the many practical problems covered are the design of trashracks and equations for the calculation of flow that varies along a pipe, which are useful in irrigation system design. The space devoted to each subject, however, is limited by the great scope of the subject matter included.

Some subjects which border hydraulics, such as hydrology and coastal engineering, which are sometimes referred to in works on hydraulics are not considered here, and certain topics such as turbulence and sediment transport which are nowadays considered to be important are treated only very briefly. Also, even in the areas where the book gives first-class coverage there are the inevitable omissions, for example siphons, culverts, jets and wakes.

In conclusion, the book is a very good compendium of useful information on the design of hydraulic systems. It will be of great use to designers because of the mass of practical data assembled, and to students because of the concise way in which each subject is treated. It also combines European and American experience more completely than previously published hydraulics handbooks such as Brater & King. However, it is not relevant to those seeking information on the nature of turbulence, sediment transport and other recondite aspects of fluid mechanics.

J. LOVELESS

Invariant Manifold Theory for Hydrodynamic Transition. By S. S. SRITHARAN. Longman, 1990. 163 pp. £17.95.

In their paper 'On the nature of turbulence' (1971) Ruelle & Takens crystallized the idea that turbulence in viscous fluid flow might somehow be the physical manifestation of a cascade of Hopf bifurcations in a dynamical system given by the equations of motion. Since then, Hopf's Bifurcation Theorem (1942) has been extended to cover infinite-dimensional dynamical systems, with an eye on possible applications to partial differential equations. A related investigation concerns subsets of phase space which, though of finite (Hausdorff) dimension and not necessarily smooth, are invariant under the flow defined by an infinite-dimensional dynamical system and capture completely its large-time asymptotic behaviour. Such sets are called global attractors. An inertial manifold is a certain type of smooth, finite-dimensional invariant manifold which contains a global attractor. One might hope to detect turbulent flows and cascades of Hopf bifurcations if such structures could be identified in the Navier–Stokes equations. In two-dimensional bounded domains the Navier–Stokes equations are known to define such an attractor. In three-dimensional domains it is not known whether attractors exist. Indeed, it is not known if the Navier–Stokes equations define a dynamical system on three-dimensional domains, and even in two-dimensions the existence of an inertial manifold has not been proved for the Navier–Stokes equations. The role of attractors for the Navier–Stokes equations therefore remains unproved.

Some believe these to be questions of only marginal interest to fluid dynamicists, but this book's author, from an aerospace engineering department, writes: *The global unique solvability problem for viscous flows in three-dimensional bounded (or unbounded) domains is now regarded as one of the profound open problems in mathematical science* (his italics!) This remark reveals a highly mathematical viewpoint. In seven brief chapters, his lecture notes give a pretty direct, concise account of the modern theory of Navier–Stokes equations, from the distributional theories of Leray (1933) and Hopf (1951), to the most recent work on the inertial manifold problem and attractors by Constantin, Foias, Ladyzhenskaya, Sell, Temam, and others (see Temam 1988). They range widely, and describe (referring to original papers for details where appropriate) the steps necessary for the proof that the two-dimensional Navier–Stokes equations on bounded domains have a compact, global attractor. Exterior and multiply connected domains, the principle of linearized stability and invariant manifold theory for Navier–Stokes are also treated in various degrees of depth. The approach is necessarily that of modern, infinite-dimensional dynamical systems theory and partial differential equations in Sobolev spaces.

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